Experiment 7
Simple and Physical Pendula

Introduction

One of Galileo's most important observations was that the period of a pendulum does not depend on the amplitude of the motion (in other words, the time to swing back and forth doesn't depend on the size of the swing). Galileo noticed this by watching chandeliars in a church, swinging due to the breeze, and he used his pulse to time the swing. We're going to less extravagent pendula, but higher tech timing devices to make some measurements on pendula.

Theory

Even a simple mass on a string is a pretty complicated object; if we keep the amplitude of the motion relatively small, though, things become a little simpler. The following expressions are only valid for a small anglular displacement of the pendulum: keep the maximum angle below about 20° to be safe.

The radius of gyration is defined by:

$$I = mk^2 \tag{1}$$

where I is the moment of inertia of the object about it's center of mass, and k is the radius of gyration. The period of a physical pendulum is, then

$$T = 2\pi\sqrt{\frac{h^2 + k^2}{gh}} \tag{2}$$

where T is the period in seconds and h is the distance between the pin and the center of mass of the object.

Squaring, and cross multiplying, you get:

$$T^2 h = \frac{4\pi^2}{g} h^2 + \frac{4\pi^2}{g} k^2. \tag{3}$$

If you plot h^2 on the x-axis and $T^2 h$ on the y-axis the slope of the (hopefully straight) line is $\frac{4\pi^2}{g}$. You can extract the acceleration of gravity from the slope. The y-axis intercept of the graph is $\frac{4\pi^2}{g} k^2$, so you can also extract the radius of gyration from your data.

For a mass on a string, $k = 0$ and $h = \ell$ the length of the string, but you should see this on you graph.

Apparatus

string, pendulum bob, meter stick with holes (measuring device and pendulum).

Procedure

1) Construct a simple pendulum by attaching a string to the bob, and hanging the mass from some support.

2) Record the time for the pendulum to swing 5-10 full cycles; divide by the number of cycles. This is the period of the pendulum. Repeat this measurement for several different amplitudes (remember to keep the angle small, though). Record the length of the pendulum.

3) Choose 5 different lengths, and repeat step 2) for each length.

4) Set up the meter stick as a physical pendulum, by placing the support pin in one of the holes.

5) Record the time for the meter stick to swing 5-10 full cycles; divide by the number of sysle. This is the period of the pendulum. Repeat this measurement for several different amplitudes. Record the distance between the pin and the center of mass of the meter stick.

6) Choose 4 different lengths and repeat step 5) for each length.

Analysis

For each pendulum length, you should have several values for the period. Calculate the average period for each length. Using the average periods for T, graph T^2h vs. h^2 and draw the best fit straight line, one graph for the simple (mass on string) and one for the physical (meter stick) pendulum. Calculate the slope of the line and compare the value of g with the accepted. Calculate the y-axis intercept and extract the value of k. How does this value compare with the table of moments of inertia in the textbook?