Experiment 14
Kirchhoff's Circuit Laws

Objective

In this experiment, we are going to study Kirchhoff's Laws for electric circuits.

Equipment

- Harrison 6200B DC Power Supply.
- Fluke 77 Multimeter.
- Two 0–999 Ω decade resistor boxes.
- One 0–9999 Ω decade resistor box.
- Assorted wires.

Part I: The Junction Rule

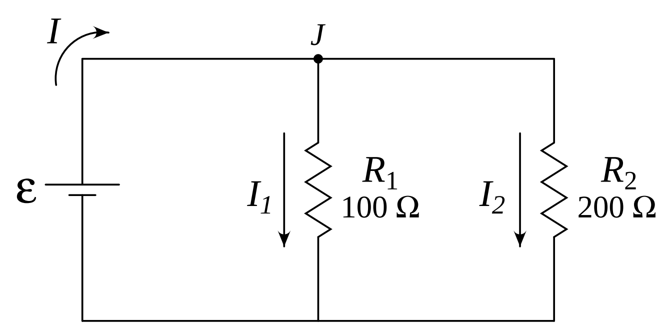
Theory

In an electric circuit, charge cannot pile up at any point (except inside capacitors), nor can charge simply disappear. As a consequence, the total current flowing into a junction in a circuit must equal the total current flowing out of that same junction:

$$\sum I_{in} = \sum I_{out}. \tag{1}$$

In this part of the lab, we will verify this rule using the simple circuit shown at the right. Applying the junction rule to point J in this circuit, we find that

$$I = I_1 + I_2. \tag{2}$$



Because the resistors are connected in parallel to each other and to the power supply, we have, from Ohm's Law:

$$\mathcal{E} = I_1 R_1 = I_2 R_2. \tag{3}$$

The second part of this equality allows us to relate I_1 and I_2:

$$I_2 = I_1 \frac{R_1}{R_2} \qquad \text{or} \qquad I_1 = I_2 \frac{R_2}{R_1}. \tag{4}$$

By using these results in Eq. (1), we may determine fraction of the total current which flows in each branch of the circuit:

$$I_1 = \frac{R_2}{R_1 + R_2} I; \qquad I_2 = \frac{R_1}{R_1 + R_2} I. \tag{5}$$

Notice that Eq. (5) implies that the current likes to follow the path of least resistance: a larger current flows in the branch containing the smaller of the two resistors.

Procedure

1. Using two of the decade resistor boxes, construct the circuit shown on page 1. Set the power supply voltage to between 6 and 7 volts. Use the multimeter as a voltmeter to verify this setting: do not trust the meter built into the power supply.

power supply voltage $\mathcal{E}$	

2. Verify that the junction rule is satisfied for this circuit. Using the 0–300 mA scale on the multimeter,* measure the current in each branch of the circuit. To do this, insert the ammeter into each branch of the circuit in turn, and record the readings.

I	
I_1	
I_2	
$I_1 + I_2$	

What is the percentage difference between your measured values of I and $I_1 + I_2$?

3. From your data, work out the ratios I_1/I and I_2/I. Compare these to the theoretical values predicted by Eq. (5).

* Don't forget that the red lead to the multimeter must be plugged into the 0–300 mA input.

4. From the power supply voltage and total current, calculate the equivalent resistance of your circuit. Compare the result to the value predicted by theory.

Part II: The Loop Rule

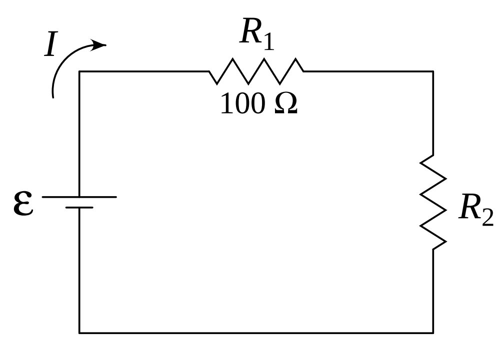
Theory

The law of conservation of energy may be applied to an electric circuit to derive Kirchhoff's Loop Rule:

$$\sum_{\text{LOOP}} \Delta V = 0. \tag{6}$$

Eq. (6) states that if you travel around *any* loop in a circuit summing all of the voltage gains and drops, the total will be zero. The sign conventions for applying the Loop Rule are:

- If you travel across a battery with emf $\mathcal{E}$ from the negative terminal to the positive terminal, write $+\mathcal{E}$.
- If you travel across a battery with emf $\mathcal{E}$ from the positive terminal to the negative terminal, write $-\mathcal{E}$.
- If you travel across a resistor in the same direction as the current is flowing, write $-IR$.
- If you travel across a resistor in the opposite direction to the current is flowing, write $+IR$.



To illustrate the Loop Rule, we will study the voltage divider circuit shown at the right. To see why we call it a voltage divider, apply the loop rule in a clockwise direction:

$$\mathcal{E} - V_1 - V_2 = 0, \tag{7}$$

where we have written V_1 for the voltage drop across the first resistor and V_2 for the voltage drop across the second resistor. According to Ohm's Law, these voltage drops are related to the current flowing in the circuit by

$$I = \frac{V_1}{R_1} = \frac{V_2}{R_2}. \tag{8}$$

Using Eq. (8) to eliminate V_1 from Eq. (7), we obtain

$$V_2 = \frac{R_2}{R_1 + R_2}\,\mathcal{E}, \tag{9}$$

that is, the power supply voltage $\mathcal{E}$ is divided in the sense that a known fraction of $\mathcal{E}$ can be picked off across R_2 if the values of the two resistances are known. Notice that the larger of the two resistors has the larger voltage drop, as is always the case when resistors are connected in series.

Procedure

5. Using two of the decade resistor boxes, construct the circuit shown on page 3. Set the power supply voltage to between 6 and 7 volts. Use the multimeter as a voltmeter to verify this setting: do not trust the meter built into the power supply.

6. Using the three values $R_2 = 10\,\Omega, 100\,\Omega$, and $500\,\Omega$ in turn, verify that the loop rule is satisfied for your circuit. Using the voltmeter function of the multimeter, measure and record the power supply voltage and the voltage drops V_1 and V_2 across the two resistors.

R_2	$10\,\Omega$	$100\,\Omega$	$500\,\Omega$
$\mathcal{E}$			
V_1			
V_2			
$V_1 + V_2$			

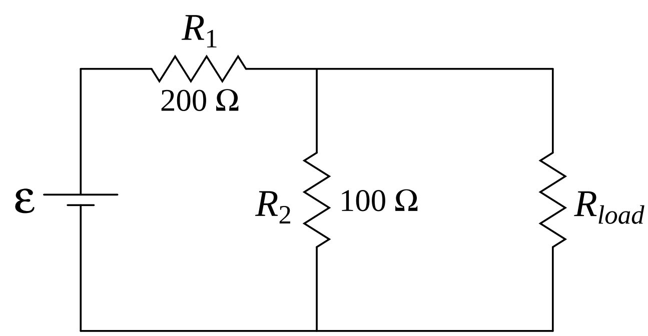
In each of the three cases, what is the percentage difference between your measured values of V and $V_1 + V_2$?

7. From your data, work out the ratio $V_2/\mathcal{E}$ in each of the three cases. Compare these to the theoretical values predicted by Eq. (9).

Part III: Loading of the Voltage Divider

Theory

The voltage divider shown on this page consists of two fixed resistors, R_1 and R_2. In the absence of any other connections (imagine that R_{load} is disconnected from the circuit) the voltage across R_2 is



$$V_2 = \frac{100\,\Omega}{100\,\Omega + 200\,\Omega}\,\mathcal{E} = \frac{\mathcal{E}}{3} \qquad (10)$$

[see Eq. (9)]. Now suppose we use this "divided" voltage to drive current into a "load," here represented by the resistor R_{load}. Some of the current which previously flowed through R_2 is now diverted into R_{load}. This effect is known as *loading* the voltage divider. To be quantitative, let us define the *loading factor*, f, by

$$f \equiv \frac{V_2(\text{with load})}{V_2(\text{without load})}. \qquad (11)$$

Eq. (11) implies that $f = 1$ when $R_{load} = \infty$ (no loading), and $f = 0$ when $R_{load} = 0$. To see how f depends on R_{load} between these limits, let $P = R_2 R_{load}/(R_2 + R_{load})$ be the equivalent resistance of the parallel combination of R_2 and R_{load}. Then, using Eq. (9) to determine the numerator and denominator of (11) we find

$$f = \frac{\dfrac{P}{R_1 + P}}{\dfrac{R_2}{R_1 + R_2}} = \frac{P(R_1 + R_2)}{R_2(R_1 + P)} = \frac{(R_1 + R_2)R_{load}}{R_1 R_2 + (R_1 + R_2)R_{load}}. \qquad (12)$$

For $R_1 = 200\,\Omega$ and $R_2 = 100\,\Omega$,

$$f = \frac{300R_{load}}{20000 + 300R_{load}}, \tag{13}$$

provided that we input the value of R_{load} in ohms. In preparation for the measurements, plot f versus D over the range $10\,\Omega$–$1000\,\Omega$ on the sheet of semi-log graph paper provided.*
Note that the result will be a gently-sloping curve, not a straight line.

Procedure

8. Using three of the decade resistor boxes, construct the circuit shown on page 5. Set the power supply voltage to between 6 and 7 volts. Use the multimeter as a voltmeter to verify this setting: do not trust the meter built into the power supply.

9. With the third decade box disconnected, measure the voltage drop across R_2. This is the value of V_2 with no load attached [the denominator of Eq. (11)].

V_2(without load)	

10. Now attach the decade box, and measure the values of V_2 for $R_{load} = 10\,\Omega$, $30\,\Omega$, $100\,\Omega$, $300\,\Omega$, and $1000\,\Omega$. You should double-check the unloaded voltage from time to time to verify that the power-supply voltage has not drifted. For each value of R_{load}, use Eq. (11) to compute your experimental value of f.

R_{load}	V_2(with load)	f
$10\,\Omega$		
$30\,\Omega$		
$100\,\Omega$		
$300\,\Omega$		
$1000\,\Omega$		

Plot your experimental results for f on the same graph that shows your calculated values. Be sure to clearly distinguish between which points are measured and which are calculated.

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* If you are unsure how to use semi-log paper, ask your instructor for help.